



## Free Assets and Their Relations with Riskless Assets

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## Abstract

Tobin's one-fund theorem states that, when a portfolio is consisting of some risky assets and a riskless asset (with return  $r_c$ ), then every efficient portfolio in the Mean-Variance optimization is a combination of the *tangency portfolio* and the riskless asset. We introduce the notion of *free asset*, which is an uncorrelated risky asset, and convert the problem for determining the tangency portfolio to a problem with lower complexity, which requires smaller portfolio, by excluding free assets with mean return  $r_c$  from initial portfolio. We show that a set of free assets, with the same mean return, can be replaced by one particular free asset with the mean return to obtain the same results. We also show that free assets (or a set of free assets) with mean return  $r_c$  and the riskless asset have a close connection and under special conditions, they almost have the same role in Mean-Variance portfolio selection problems.

*Keywords:* Mean-Variance optimization; efficient frontier; tangency portfolio; one-fund theorem.

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## 1 Introduction

The seminal work on Modern Portfolio Theory was originated by Markowitz [1] when he presented his Mean-Variance (M-V) portfolio selection approach. It is a single-period investment theory in a

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