PROBLEM OF THE WEEK Solution of Problem No. 7 (Fall 2013 Series)

Problem:

Find an explicit one to one correspondence between [0, 2013] and (0, 2013].

Solution: (by Indira Iyer, Junior, Computer Engineering, Purdue University)

An explicit bijection

$$f:[0,2013]\longrightarrow (0,2013]$$

can be given as,

$$f(x) = \begin{cases} 2013 & \text{if } x = 0\\ x & \text{if } x \in \left(\frac{1}{2^{n+1}}2013, \frac{1}{2^n}2013\right) & \text{for some } n \in \mathbb{N} \cup \{0\}\\ \frac{2013}{2^{n+1}} & \text{if } x = \frac{2013}{2^n} & \text{for some } n \in \mathbb{N} \cup \{0\}. \end{cases}$$

The problem was also solved by:

<u>Undergraduates</u>: Bennett Marsh (Jr. Phys & Math.)

<u>Graduates</u>: Tairan Yuwen (Chemistry), Samson Zhou (CS)

Others: Charles Burnette (Grad Student, Drexel Univ.), Hongwei Chen (Professor, Christopher Newport Univ, Virginia), Hubert Desprez (Paris, France), Paul Farias (W. Lafayette, IN), Massimo Frittelli (Italy), Andrew Garmon (Christopher Newport University alumni), Elie Ghosn (Montreal, Quebec), Joe Klobusicky (Graduate student, Brown Univ.), Peter Kornya (Retired Faculty, Ivy Tech), Oliver Kroll (San Francisco, CA), Steven Landy (Physics Faculty, IUPUI), Yun-chen Pan & Wei-Xiang Lien (Miaoli, Taiwan), Vladimir B. Lukianov (Lecturer, Tel-Aviv), Jean Pierre Mutanguha (Student, Oklahoma Christian Univ), Paolo Perfetti (Roma, Italy), Achim Roth (Data Protection Officer, Germany), Sorin Rubinstein (TAU faculty, Tel Aviv, Israel), Craig Schroeder (Postdoc. UCLA), Rick Shilling (Orlando, FL), David Stoner (HS Student, Aiken, S. Carolina), Aaron Tang (Student, National Univ. of Singapore), Justin Wolfe (Grad. student, Old Dominion Univ. VA)