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Comments on "Strong convergence rates for backward Euler on a class of nonlinear jump–diffusion problems" [J. Comput. Appl. Math. 205 (2007) 949–956]

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ABSTRACT

In this note, we correct an inaccuracy of Eq. (13) in the proof of Theorem 1 in the paper (Higham and Kloeden, 2007). We also give a complete proof of the theorem by applying the Burkholder–Davis–Gundy Inequality.

1. Introduction

We first explain the inaccuracy in [1, Theorem 1].

To prove Theorem 1, based on [1, Eq. (12)], for a martingale $M(t)$, they have

$$|e(t)|^2 \leq K \int_0^t \mathbb{E}\left[\left|e(s^-)\right|^r\right] \mathrm{d}s + K\Big(\sup_{0\leq s\leq t} \left|\bar{Y}_{s-} - Y_{s-}\right|^2\Big) \int_0^t \left(1 + \left|\bar{Y}_{s-}\right|^q + |Y_{s-}|^q\right) \mathrm{d}s + M(t). \tag{1}$$

Since $\sup_{0\leq s\leq t} \left|\bar{Y}_{s-} - Y_{s-}\right|^2$ is not independent of $\int_0^t (1+\left|\bar{Y}_{s-}\right|^q+|Y_{s-}|^q)\mathrm{d}s$, we need to apply the Cauchy–Schwarz inequality to take the expectation. This results

$$\begin{aligned}
\mathbb{E}\left[\Big(\sup_{0\leq s\leq t} \left|\bar{Y}_{s-} - Y_{s-}\right|^2\Big) \int_0^t \left(1 + \left|\bar{Y}_{s-}\right|^q + |Y_{s-}|^q\right)\mathrm{d}s\right] &\leq \left(\mathbb{E}\left[\sup_{0\leq s\leq t} \left|\bar{Y}_{s-} - Y_{s-}\right|^2\right]^2\right)^{\frac{1}{2}} \left(\mathbb{E}\left[\int_0^t (1 + \left|\bar{Y}_{s-}\right|^q + |Y_{s-}|^q)\mathrm{d}s\right]^2\right)^{\frac{1}{2}} \\
&\leq \sqrt{t}\left(\mathbb{E}\left[\sup_{0\leq s\leq t} \left|\bar{Y}_{s-} - Y_{s-}\right|^4\right]\right)^{\frac{1}{2}} \left(\int_0^t \mathbb{E}\left[1 + \left|\bar{Y}_{s-}\right|^q + |Y_{s-}|^q\right]^2 \mathrm{d}s\right)^{\frac{1}{2}},
\end{aligned} \tag{2}$$

then one need to calculate $\mathbb{E}\left[\sup_{0\leq s\leq t} \left|\bar{Y}_{s-} - Y_{s-}\right|^4\right]$ instead of power 2, they have wrongly utilized in [1, Eq. (13)].

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