

PROBLEM OF THE WEEK
Solution of Problem No. 4 (Fall 2013 Series)

Problem:

Let $a, b > 0$. Find the curve in the first quadrant which passes through (a, b) and has the property that if the tangent line is drawn at any point p on the curve then that part of this tangent line which lies in the first quadrant is bisected at p .

**This problem was proposed by Tom Engelsman C.I.T. Tampa, FL.

Solution: (by Bennett Marsh, Junior, Physics/Math, Purdue University)

Let (x, y) be in the first quadrant. If we were to draw a line through this point such that the portion of the line in the first quadrant was bisected at (x, y) , then the x - and y -intercepts of the line would have to be $x_{int} = 2x$ and $y_{int} = 2y$. The slope of this line is then just $m = -y_{int}/x_{int} = -y/x$. This means that the desired curve must satisfy the differential equation

$$\frac{dy}{dx} = -\frac{y}{x}.$$

Integrating, this leads to

$$y = \frac{c}{x},$$

and plugging in the condition $y(a) = b$, we find

$$y = \frac{ab}{x}.$$

The problem was also solved by:

Undergraduates: Rustam Orazaliyev (Jr. Actuarial Sci), Jingbo Wu (So. Tech.)

Graduates: Sambit Palit (ECE), Tairan Yuwen (Chemistry), Samson Zhou (CS)

Others: Charles Burnette (Grad Student, Drexel Univ.), Pierre Castelli (Antibes, France), Pawan Chawla (CA), Hongwei Chen (Professor, Christopher Newport Univ., Virginia), Mark Crawford Jr. (Professor, Waubensee Community College, IL), Hubert Desprez (Paris, France), Tom Engelsman (Tampa, FL), Massimo Frittelli (Italy), Andrew Garmon (Christopher Newport University alumni), Elie Ghosn (Montreal, Quebec), Gaoyue and Gaopeng Guo (Students, Ecole Polytechnique, France), Kipp Johnson (Valley Catholic HS)

teacher, Oregon), Peter Kornya (Retired Faculty, Ivy Tech), Steven Landy (Physics Faculty, IUPUI), Wei-Xiang Lien (Miaoli, Taiwan), Dimitris Los (Athens, Greece), Jean Pierre Mutanguha (Student, Oklahoma Christian Univ.), Achim Roth (Data Protection Officer, Germany), Sorin Rubinstein (TAU faculty, Tel Aviv, Israel), K. Sambath, Jason L. Smith (Professor, Richland Community College, IL), David Stigant, David Stoner (HS Student, Aiken, S. Carolina), Bharath Swaminathan (Caterpillar, India), Aaron Tang (Student, National Univ. of Singapore), Motohiro Tsuchiya (Graduate student, Bethesda, MD)