# Multilevel Monte-Carlo Simulation Applied to Lévy Driven Assets

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**Abstract**

Inspired by recent advances in the application of the Multilevel Monte-Carlo (MLMC) approach to Lévy driven assets, we benefit this method to price European and Asian options for the Merton jump-diffusion model. Recently, Belomestny and Nagapetyan have introduced the weak MLMC scheme which allow us to use weak approximation methods with the MLMC algorithm. Our contribution in this work is to extend their result to $L_p(\Omega), p \geq 2$, spaces. Additionally, we use weak Euler scheme to numerically estimate the asset and apply weak MLMC method to price the option.

**Keywords:** Multilevel Monte-Carlo method, Lévy process, price estimation

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## 1 Introduction

Introduced by M. Giles [3] the MLMC method is a way to efficiently distribute the computational complexity caused by the variance and the bias over a series of levels. In contrast, the standard Monte-Carlo (MC) method deals with these two problems at the same time.

To show the idea assume a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ in which we have an $n$-dimensional process $(X_t)_{t\geq 0}$, which solves the following Lévy driven stochastic differential equation (SDE)

$$\mathrm{d}X_t = a(X_{t^-})\mathrm{d}Z_t, \quad 0 \leq t \leq T, \tag{1}$$

with $X_0$ being a known $\mathbb{R}^n$-valued random variable, $Z_t = (Z_{t,1}, \ldots, Z_{t,q})$, $t \geq 0$ is a $q$-dimensional Lévy process and the mapping $a : \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^q$ is Lipschitz continuous and satisfies the linear growth condition such that the solution exists and is unique [1].

We are not interested in the solution of (1) but rather the expectation of a functional, i.e.

$$P = \mathbb{E}[f(X_T)], \tag{2}$$

which gives the fair price of the option based on the Lipschitz continuous payoff $f$ on the asset $X$, or for short, just the price [2]. Using the numerical approximation $X_t^{\Delta}$ with stepsize $\Delta = T/2^L$ instead and expanding (2) into a telescopic sum we obtain:

$$\begin{aligned} \tilde{P}^{\Delta} &= \mathbb{E}[f(X_T^T)] && (3)\\ &+ \mathbb{E}\left[f(X_T^{T/2}) - f(X_T^T)\right] && (4)\\ &+ \mathbb{E}\left[f(X_T^{T/4}) - f(X_T^{T/2})\right] \\ &\vdots \\ &+ \mathbb{E}\left[f(X_T^{\Delta}) - f(X_T^{2\Delta})\right]. && (5) \end{aligned}$$

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